EPJ Web of Conferences **118**, 01012 (2016) DOI: 10.1051/epjconf/201611801012
FCCP2015

Radiative τ leptonic decays and the possibility to determine the τ dipole moments

Matteo Fael[1,a]

[1] *Albert Einstein Center for Fundamental Physics, Institute for Theoretical Physics, University of Bern, CH-3012 Bern, Switzerland*

Abstract. Precise data on radiative leptonic τ decays offer the opportunity to probe the electromagnetic properties of the τ and may allow to determine its anomalous magnetic moment which, in spite of its precise Standard Model prediction, has never been measured. Recently, the branching fractions of the radiative leptonic τ decays ($\tau \to l\nu\bar{\nu}\gamma$, with $l = e, \mu$) were measured by the BaBar collaboration. These precise measurements, with a relative error of about 3%, must be compared with the branching ratios at the next-to-leading order in QED. Indeed the radiative corrections are expected to be of order 10%, for $l = e$, and 3%, for $l = \mu$. Here, we present the prediction of the differential decay rates and branching ratios of the radiative leptonic τ decays in the Standard Model at the next-to-leading order and we compare them with the recent BaBar measurements. Moreover, we report on a dedicated feasibility study for the measurements of the τ anomalous magnetic moment at Belle and Belle II.

1 Introduction

The electric dipole moment (EDM) and the anomalous magnetic moment ($g-2$) of a lepton are physical observables sensitive to quantum corrections induced by virtual particles that populate the vacuum. For this reason they are very well suited to test the Standard Model (SM) of particle physics and to unveil unknown new physics hidden at high energy. The electron and muon $g-2$ have been measured with the wonderful precision of 0.24 ppb [1] and 540 ppb [2], and they agree with the SM predictions at the level of ~ 1.4 and ~ 3.8 standard deviations, respectively [3]. In contrast, the short lifetime of the τ lepton (2.9×10^{-13} s) poses many difficulties for the experimental determination of its dipole moments and indeed it has so far prevented the direct measurement of the $g-2$ by means of the τ spin precession in a magnetic field, like in the electron and muon $g-2$ experiments. In fact, the present bound on the τ $g-2$ is only of $O(10^{-2})$, more than an order of magnitude bigger than the leading contribution $\alpha/(2\pi) \approx 0.001$ – result obtained long ago by Schwinger [4]. Therefore, experiments must attempt the extraction of indirect bounds from τ pair production and decays by comparing sufficiently precise data with the SM predictions.

We propose to measure the electromagnetic dipole moments of the τ lepton via radiative leptonic τ decays ($\tau \to l\gamma\nu\bar{\nu}$ with $l = \mu, e$) by means of an effective Lagrangian approach. Radiative τ leptonic decays can indeed be predicted with very high precision and can be formulated in term of the Bouchiat-Michel-Kinoshita-Sirlin parameters [5–8].

Moreover, the BaBar collaboration performed recently the measurements of the $\tau \to l\gamma\nu\bar{\nu}$ branching fractions for a minimum photon energy $\omega_0 = 10$ MeV in the τ rest frame [9]. The experimental precision of these measurements, around 3%, requires the SM prediction of the branching ratios at next-to-leading order (NLO). Indeed these radiative corrections are expected to be of relative order $(\alpha/\pi)\ln(m_l/m_\tau)\ln(\omega_0/m_\tau)$, corresponding to a large 10% correction for $l = e$, and 3% for $l = \mu$.

After establishing our conventions in sec. 2 and introducing an effective Lagrangian for the study of the τ dipole moments, we briefly review in secs. 3 and 4 the present theoretical and experimental status on the τ $g-2$ and EDM. The SM decay rates and branching ratios at NLO, and their comparison with recent BaBar measurements, are presented in sec. 5. In sec. 6 we report the achievable sensitivity to the τ electromagnetic moments at Belle II experiment. Conclusions are drawn in sec. 7.

2 The τ lepton electromagnetic form factors

The most general vertex function describing the interaction between initial and final states of an on-shell τ lepton, with four-momenta p and p' respectively, and a photon can be written in the form

$$\Gamma^\mu(q^2) = ie\left\{\gamma^\mu F_1(q^2) + \frac{\sigma^{\mu\nu}q_\nu}{2m_f}\left[iF_2(q^2) - F_3(q^2)\gamma_5\right] + \left(\gamma^\mu - \frac{2q^\mu m_f}{q^2}\right)\gamma_5 F_4(q^2)\right\}, \tag{1}$$

[a] e-mail: fael@itp.unibe.ch

where $e > 0$ is the positron charge, m_τ the mass of the τ, $\sigma_{\mu\nu} = i/2\,[\gamma_\mu, \gamma_\nu]$ and $q = p' - p$ is the ingoing four-momentum of the off-shell photon. The functions $F_2(q^2)$ and $F_3(q^2)$ are related, in the limit $q^2 \to 0$, to the measurable quantities

$$F_2(0) = a_\tau, \qquad F_3(0) = d_\tau \, \frac{2m_\tau}{e}, \tag{2}$$

where a_τ and d_τ are the anomalous magnetic moment and electric dipole moment of the τ, respectively.

Deviations of the τ dipole moments from the SM values can be analyzed in the framework of dimension-six gauge-invariant operators. Out of the complete set of 59 independent gauge invariant operators in [10, 11], only two of them can directly contribute to the τ $g-2$ and EDM at tree level (i.e., not through loop effects):

$$Q^{33}_{lW} = \left(\bar{l}_\tau \sigma^{\mu\nu} \tau_R\right) \sigma^I \varphi \, W^I_{\mu\nu}, \tag{3}$$

$$Q^{33}_{lB} = \left(\bar{l}_\tau \sigma^{\mu\nu} \tau_R\right) \varphi \, B_{\mu\nu}, \tag{4}$$

where φ and $l_\tau = (\nu_\tau, \tau_L)$ are the Higgs, and the left-handed SU(2) doublets, σ^I the Pauli matrices, $W^I_{\mu\nu}$ and $B_{\mu\nu}$ are the gauge field strength tensors. The leading non-standard effective Lagrangian relevant for our study is therefore given by

$$\mathcal{L}_{\text{eff}} = \frac{1}{\Lambda^2} \left[C^{33}_{lW} Q^{33}_{lW} + C^{33}_{lB} Q^{33}_{lB} + \text{h.c.} \right]. \tag{5}$$

After the electroweak symmetry breaking, the two operators mix and give additional, beyond the SM, contributions to the τ anomalous magnetic moment and EDM:

$$\tilde{a}_\tau = \frac{2m_\tau}{e} \frac{\sqrt{2}v}{\Lambda^2} \, \text{Re} \left[\cos\theta_{\text{W}} C^{33}_{lB} - \sin\theta_{\text{W}} C^{33}_{lW} \right], \tag{6}$$

$$\tilde{d}_\tau = \frac{\sqrt{2}v}{\Lambda^2} \, \text{Im} \left[\cos\theta_{\text{W}} C^{33}_{lB} - \sin\theta_{\text{W}} C^{33}_{lW} \right], \tag{7}$$

where $v = 246$ GeV. Deviations of the τ dipole moments from the SM values could be then determined, possibly down to the level of $O(10^{-3})$, via precise data on τ pair production and τ decays.

3 Status of the τ lepton $g-2$ and EDM

The SM prediction for a_τ is given by the sum of QED, hadronic and electroweak (EW) terms. The QED contribution has been computed up to three loops: $a_\tau^{\text{QED}} = 117\,324\,(2) \times 10^{-8}$ [12], where the uncertainty $\pi^2 \ln^2(m_\tau/m_e)(\alpha/\pi)^4 \sim 2 \times 10^{-8}$ has been assigned for uncalculated four-loop contributions. The errors due to the uncertainties of the $O(\alpha^2)$ and $O(\alpha^3)$ terms, as well as that induced by the uncertainty of α, are negligible. The sum of the one- and two-loop EW contributions is $a_\tau^{\text{EW}} = 47.4\,(5) \times 10^{-8}$ [13–16]. The uncertainty encompasses the estimated errors induced by hadronic loop effects, neglected two-loop bosonic terms and the missing three-loop contribution. It also includes the tiny errors due to the uncertainties in M_{top} and m_τ.

Similarly to the case of the muon $g-2$, the leading-order hadronic contribution to a_τ is obtained via a dispersion integral of the total hadronic cross section of the e^+e^- annihilation (the role of low energies is very important, although not as much as for a_μ). The result of the latest evaluation, using the whole bulk of experimental data below 12 GeV, is $a_\tau^{\text{HLO}} = 337.5\,(3.7) \times 10^{-8}$ [16]. The hadronic higher-order (α^3) contribution a_τ^{HHO} can be divided into two parts: $a_\tau^{\text{HHO}} = a_\tau^{\text{HHO}}(\text{vp}) + a_\tau^{\text{HHO}}(\text{lbl})$. The first one, the $O(\alpha^3)$ contribution of diagrams containing hadronic self-energy insertions in the photon propagators, is $a_\tau^{\text{HHO}}(\text{vp}) = 7.6\,(2) \times 10^{-8}$ [17]. Note that naïvely rescaling the corresponding muon $g-2$ result by a factor m_τ^2/m_μ^2 leads to the incorrect estimate $a_\tau^{\text{HHO}}(\text{vp}) \sim -28 \times 10^{-8}$ (even the sign is wrong!). Estimates of the light-by-light contribution $a_\tau^{\text{HHO}}(\text{lbl})$ obtained rescaling the corresponding one for the muon $g-2$ by a factor m_τ^2/m_μ^2 fall short of what is needed – this scaling is not justified. The parton-level estimate of [16] is $a_\tau^{\text{HHO}}(\text{lbl}) = 5(3) \times 10^{-8}$, a value much lower than those obtained by naïve rescaling. Adding up the above contributions one obtains the SM prediction [16]

$$a_\tau^{\text{SM}} = a_\tau^{\text{QED}} + a_\tau^{\text{EW}} + a_\tau^{\text{HLO}} + a_\tau^{\text{HHO}} = 117\,721\,(5) \times 10^{-8}. \tag{8}$$

Errors were added in quadrature.

The EDM interaction violates the discrete CP symmetry. In the SM, with massless neutrinos, the only source of CP violation is the CKM-phase (and a possible θ-term in QCD sector). It can be shown [18, 19] that all CP-violating amplitudes are proportional to the Jarlskog invariant J defined via the convention-invariant equation [19]

$$\text{Im}\left[V_{ij} V_{kl} V^*_{il} V^*_{kj} \right] = J \sum_{m,n} \varepsilon_{ikm} \varepsilon_{jln} \,. \tag{9}$$

Therefore, a fundamental lepton EDM must arise from virtual quarks linked to the lepton through the W boson and also be sensitive to the imaginary part of the V_{CKM} matrix elements. The leading contribution is naïvely expected at the three-loop level, since two-loop diagram is proportional to $|V_{ij}|^2$. The problem was first analyzed in some detail in [20], but it was subsequently shown that the three-loop diagrams also yield a zero EDM contribution in the absence of gluonic corrections to the quark lines [21]. For this reason, lepton EDMs are predicted to be extremely small in the SM, of the $O(10^{-38} - 10^{-35})\,e{\cdot}\text{cm}$ [22], which is far below the current experimental capabilities. Indeed, present experiments can only probe $d_\tau \sim O(10^{-17})\,e{\cdot}\text{cm}$. Also for the electron, $d_e^{\text{exp}} < 0.87 \times 10^{-28} e{\cdot}\text{cm}$ [23] while $d_e^{\text{SM}} \sim O(10^{-38})\,e{\cdot}\text{cm}$ – it is hard to imagine improvements in the sensitivity by ten orders of magnitude! However, new EDM effects could arise at one or two loop from new physics that violates P and T, and be much larger than the tiny SM prediction even if they come from high mass scales. They generally induce large contributions to lepton and neutron EDMs [24], and although there has been no experimental evidence for an EDM so far, there is considerable hope to gain new insights into the nature of CP violation through this kind of experiments.

4 Experimental determination

The present resolution on the τ anomalous magnetic moment is only of $O(10^{-2})$ [25], more than an order of magnitude larger than its SM prediction in Eq. (8). In fact, while the SM value of a_τ is known with a tiny uncertainty of 5×10^{-8}, the τ short lifetime has so far prevented the determination of a_τ by measuring the τ spin precession in a magnetic field, like in the electron and muon g–2 experiments. The present PDG limit on the τ g–2 was derived in 2004 by the DELPHI collaboration from $e^+e^- \to e^+e^-\tau^+\tau^-$ total cross section measurements at $\sqrt{s}$ between 183 and 208 GeV at LEP2 (the study of a_τ via this channel was proposed in [26]). The measured values of the cross-sections were used to extract limits on the τ g–2 by comparing them to the SM values, assuming that possible deviations were due to non-SM contributions to a_τ. The obtained limit at 95% CL is [25]

$$-0.052 < \tilde{a}_\tau < 0.013, \tag{10}$$

which can be also expressed in the form of central value and error as [25]

$$\tilde{a}_\tau = -0.018\,(17). \tag{11}$$

The present PDG limit on the EDM of the τ lepton at 95% CL is

$$\begin{aligned} -2.2 < \mathrm{Re}(\tilde{d}_\tau) < 4.5 \;\; (10^{-17}\; e\cdot\mathrm{cm}), \\ -2.5 < \mathrm{Im}(\tilde{d}_\tau) < 0.8 \;\; (10^{-17}\; e\cdot\mathrm{cm}); \end{aligned} \tag{12}$$

it was obtained by the Belle collaboration [27] following the analysis of ref. [28] for the impact of an effective operator for the τ EDM in the process $e^+e^- \to \tau^+\tau^-$.

The reanalysis of ref. [29] of various LEP and SLD measurements – mainly of the $e^+e^- \to \tau^+\tau^-$ cross sections – allowed the authors to set the indirect 2σ confidence interval

$$-0.007 < \tilde{a}_\tau < 0.005, \tag{13}$$

a bound stronger than that in Eq. (10). This analysis assumed $\tilde{d}_\tau = 0$.

At the LHC, bounds on the τ dipole moments are expected to be set in τ pair production via Drell-Yan [30, 31] or double photon scattering processes [32]. The best limits achievable in $pp \to \tau^+\tau^- + X$ are estimated to be comparable with present existing ones if one assumes that the total cross section for τ pair production will be measured at the 14% level. Earlier proposals can be found in [33, 34].

Another proposed method to determine the $\tilde{a}_\tau$ would use the channeling of polarized τ leptons in a bent crystal similarly to the suggestion for the measurement of magnetic moments of short-living baryons [35]. This method has been successfully tested by the E761 collaboration at Fermilab, that measured the magnetic moment of the Σ^+ hyperon [36]. However the challenge with this method is to produce a polarized beam of τ leptons; the decay $B^+ \to \tau^+\nu_\tau$ could provide such polarized τ [37] but it has a very tiny branching ratio of $O(10^{-4})$.

The Belle II experiment at the upcoming high-luminosity B factory SuperKEKB [38] will offer new opportunities to improve the determination of the τ electromagnetic properties. The authors of ref. [39] proposed to determine the Pauli form factor $F_{2V}(q^2)$ of the τ via $\tau^+\tau^-$ production in e^+e^- collisions at the Υ resonances (Υ(1S), Υ(2S) and Υ(3S)) with a sensitivity of $O(10^{-5})$ or even better. The center-of-mass energy at super B factories is $\sqrt{s} \sim M_{\Upsilon(4S)} \approx 10$ GeV, so that the form factor $F_{2V}(q^2)$ is no longer the anomalous magnetic moment. Furthermore, the contributions to the $e^+e^- \to \tau^+\tau^-$ cross section arise not only from the usual s-channel one-loop vertex corrections, but also from box diagrams, which should be somehow subtracted out. The strategy proposed in [39] to eliminate the contamination from the box diagrams has been to measure the observables on top of the Υ resonances: in this kinematic regime the (non-resonant) box diagrams are numerically negligible and only one-loop corrections to the $\gamma\tau\tau$ vertex are relevant.

However, it is very difficult to resolve the narrow peaks of the $\Upsilon(1S, 2S, 3S)$ in the $\tau^+\tau^-$ decay channel – the $\Upsilon(4S)$ decays almost entirely in $B\bar{B}$ – because of the natural irreducible beam energy spread associated to any e^+e^- synchrotron. If we compare the total width for the Υ resonances ($\Gamma^{\Upsilon}_{\mathrm{tot}} \sim 20 - 50$ keV) with the SuperKEKB beam energy spread $\sigma_{\mathrm{w}} = 5.45$ MeV [40], we note that at the Belle II the $\tau^+\tau^-$ events produced with beams at a centre of mass energy $\sqrt{s} \sim M_\Upsilon$ are mostly due to non-resonant interaction. The situation at Belle was similar (the energy spread at KEKB was $\sigma_{\mathrm{w}} = 5.24$ MeV [41]). Eventually, the measurement of the $e^+e^- \to \tau^+\tau^-$ cross section on top of the Υ resonances will not eliminate the contamination of the box diagrams.

5 Radiative τ leptonic decays: theoretical framework

In the next two sections we propose a new method to determine the electromagnetic dipole moments of the τ via measurements of its radiative leptonic decays. The SM prediction, at NLO, for the differential rate of the radiative leptonic decays

$$\tau^\pm \to l^\pm\, \nu\, \bar{\nu}\, \gamma, \tag{14}$$

with $l = e$ or μ, of a polarized $\tau^\pm$ with mass m_τ in its rest frame is

$$\begin{aligned} \frac{d^6\Gamma^\pm(y_0)}{dx\,dy\,d\Omega_l\,d\Omega_\gamma} &= \frac{\alpha\, G_F^2 m_\tau^5}{(4\pi)^6}\,\frac{x\beta_l}{1+\delta_{\mathrm{w}}(m_\mu, m_e)} \times \\ &\times \Big[G \mp x\beta_l\,\hat{n}\cdot\hat{p}_l\,J \mp y\,\hat{n}\cdot\hat{p}_\gamma\,K + xy\beta_l\,\hat{n}\cdot\left(\hat{p}_l\times\hat{p}_\gamma\right)L\Big], \end{aligned} \tag{15}$$

where $G_F = 1.166\,378\,7(6) \times 10^{-5}$ GeV^{-2} [42] is the Fermi constant, defined from the muon lifetime, and $\alpha = 1/137.035\,999\,157\,(33)$ is the fine-structure constant [43, 44]. Calling m the mass of the final charged lepton (neutrinos and antineutrinos are considered massless) we define $r = m/m_\tau$ and $r_{\mathrm{w}} = m_\tau/M_{\mathrm{w}}$, where M_{w} is the W-boson mass; p and $n = (0, \hat{n})$ are the four-momentum and polarization vector of the initial τ, with $n^2 = -1$ and $n \cdot p = 0$. Also, $x = 2E_l/m_\tau$, $y = 2E_\gamma/m_\tau$ and $\beta_l \equiv |\vec{p}_l|/E_l = \sqrt{1-4r^2/x^2}$, where $p_l = (E_l, \vec{p}_l)$ and $p_\gamma = (E_\gamma, \vec{p}_\gamma)$ are the four-momenta of the final charged lepton and photon, respectively. The final charged lepton and photon are emitted at solid angles Ω_l and Ω_γ, with normalized three-momenta $\hat{p}_l$ and $\hat{p}_\gamma$, and c is the cosine of the angle between $\hat{p}_l$ and $\hat{p}_\gamma$. The term $\delta_{\mathrm{W}}(m_\mu, m_e) = 1.04 \times 10^{-6}$ is the tree-level correction to muon decay induced by the W-boson propagator [45, 46]. Equation (15) includes the possible emission of an additional soft photon with normalized energy y' lower than the detection threshold $y_0 \ll 1$: $y' < y_0 < y$. The function $G(x, y, c; y_0)$ and, analogously, J and K, are given by

$$G(x, y, c, y_0) = \\ = \frac{4}{3yz^2}\left[g_0(x, y, z) + r_{\mathrm{W}}^2\, g_{\mathrm{W}}(x, y, z) + \frac{\alpha}{\pi}\, g_{\mathrm{NLO}}(x, y, z, y_0)\right], \tag{16}$$

where $z = xy(1 - c\beta_l)/2$; the LO function $g_0(x, y, z)$, computed in [47–50], arises from the pure Fermi V–A interaction, whereas $g_{\mathrm{W}}(x, y, z)$ is the leading contributions of the W-boson propagator derived in [46]. The NLO term $g_{\mathrm{NLO}}(x, y, z, y_0)$ is the sum of the virtual and soft bremsstrahlung contributions calculated in [51] (see also refs. [52, 53]). The function $L(x, y, z)$, appearing in front of the product $\hat{n} \cdot \left(\hat{p}_l \times \hat{p}_\gamma\right)$, does not depend on y_0; it is only induced by the loop corrections and is therefore of $O(\alpha/\pi)$. The (lengthy) explicit expressions of G, J, K and L are provided in [51]. If the initial $\tau^\pm$ are not polarized, eq. (15) simplifies to

$$\frac{d^3\Gamma(y_0)}{dx\, dc\, dy} = \frac{\alpha G_F^2 m_\tau^5}{(4\pi)^6} \frac{8\pi^2\, x\beta_l}{1 + \delta_{\mathrm{W}}(m_\mu, m_e)}\, G(x, y, c, y_0). \tag{17}$$

At the LO, the analytic integration over the allowed kinematic ranges leads, for a minimum photon energy $y_0 = 2\omega_0/m_\tau$, to [49, 54]

$$\Gamma_{\mathrm{LO}}(y_0) = \frac{G_F^2 m_\tau^5}{192\pi^3} \frac{\alpha}{3\pi}\, H(y_0), \tag{18}$$

$$H(y_0) = 3\,\mathrm{Li}_2(y_0) - \frac{\pi^2}{2} - \frac{1}{2}\left(6 + \bar{y}_0^3\right)\bar{y}_0 \ln \bar{y}_0 + \\ + \left(\ln r + \frac{17}{12}\right)\left(6 \ln y_0 + 6\bar{y}_0 + \bar{y}_0^4\right) + \\ + \frac{1}{48}\left(125 + 45 y_0 - 33 y_0^2 + 7 y_0^3\right)\bar{y}_0, \tag{19}$$

where $\bar{y}_0 = 1 - y_0$ and the dilogarithm is defined by $\mathrm{Li}_2(z) = -\int_0^z dt\, \frac{\ln(1-t)}{t}$. Terms depending on the mass ratio r have been neglected in the expression for $H(y_0)$, with the obvious exception of the logarithmic contribution which diverges in the limit $r \to 0$. However, terms in the integrand $G_{\mathrm{LO}}(x, y, c)$ proportional to r^2 were not neglected when performing the integral to obtain (19), as they lead to terms of $O(1)$ in the integrated result $H(y_0)$. This feature, first noted in [55], is due to the appearance of right-handed electrons and muons in the final states of (14) even in the limit $r \to 0$, and is a consequence of helicity-flip bremsstrahlung in QED [55–58].

At the NLO, which allows for double photon emission, the branching ratios of the radiative decays (14) can be distinguished in two types due to the double real emission:

- The "inclusive" branching ratio, $\mathcal{B}^{\mathrm{Inc}}(y_0)$, where in the final state there is at least one photon with energy higher than y_0.
- The "exclusive" branching ratio, $\mathcal{B}^{\mathrm{Exc}}(y_0)$, where in the final state there is one, and only one, photon of energy larger than the detection threshold y_0.

It is clear that at the LO the theoretical predictions for these exclusive and inclusive branching ratios coincide – double bremsstrahlung events are simply not considered.

Exclusive and inclusive branching ratios for the radiative decays (14) were computed, for a threshold $\omega_0 = y_0\,(m_\tau/2) = 10$ MeV, in ref. [51] and are reported in table 1. Uncertainties were estimated for uncomputed NNLO corrections, numerical errors, and the experimental errors of the lifetimes. The former were estimated to be $\delta\mathcal{B}_{\mathrm{NLO}}^{\mathrm{Exc/Inc}} \sim (\alpha/\pi) \ln r \ln(\omega_0/m_\tau)\, \mathcal{B}_{\mathrm{NLO}}^{\mathrm{Exc/Inc}}$. For $\omega_0 = 10$ MeV they are about 10% and 3% for $\tau \to e\bar{\nu}\nu\gamma$ and $\tau \to \mu\bar{\nu}\nu\gamma$, respectively. They appear with the subscript "N" in table 1. Numerical errors, labeled by the subscript "n", are smaller than those induced by missing radiative corrections. These two kinds of uncertainties were combined to provide the theoretical error of the final $\mathcal{B}^{\mathrm{Exc}}$ and $\mathcal{B}^{\mathrm{Inc}}$ predictions, labeled by the subscript "th". The uncertainty due to the experimental error of the lifetimes is labeled by the subscript "τ".

The recent measurements by the BaBar collaboration of the branching ratios of the radiative decays $\tau \to l\bar{\nu}\nu\gamma$, with $l = e$ and μ, for a minimum photon energy $\omega_0 = 10$ MeV in the τ rest frame, are [9]:

$$\mathcal{B}_{\mathrm{EXP}}(\tau \to e\bar{\nu}\nu\gamma) = 1.847\,(15)_{\mathrm{st}}(52)_{\mathrm{sy}} \times 10^{-2}, \tag{20}$$

$$\mathcal{B}_{\mathrm{EXP}}(\tau \to \mu\bar{\nu}\nu\gamma) = 3.69\,(3)_{\mathrm{st}}(10)_{\mathrm{sy}} \times 10^{-3}, \tag{21}$$

where the first error is statistical and the second is systematic. These results are substantially more precise than the previous measurements of the Cleo collaboration [59]. The experimental values in eqs. (20,21) must be compared with our predictions for the exclusive branching ratios in table 1. For $\tau \to \mu\bar{\nu}\nu\gamma$ decays, the branching ratio measurement and prediction agree within 1.1 standard deviations (1.1σ). On the contrary, the experimental and theoretical values for $\tau \to e\bar{\nu}\nu\gamma$ decays differ by $2.02\,(57) \times 10^{-3}$, i.e. by 3.5σ. This puzzling discrepancy deserves further researches.

6 τ dipole moments via $\tau \to l\gamma\nu\bar{\nu}$ decays

The effective Lagrangian (5) generates additional nonstandard contributions to the differential decay rate in eq. (15). For a $\tau^\pm$ they can be summarised in the shifts

$$G \to G + \tilde{a}_\tau\, G_a, \qquad J \to J + \tilde{a}_\tau\, J_a, \tag{22}$$

$$K \to K + \tilde{a}_\tau\, K_a, \qquad L \to L \mp (m_\tau/e)\, \tilde{d}_\tau\, L_d. \tag{23}$$

Tiny terms of $O(\tilde{a}_\tau{}^2)$, $O(\tilde{d}_\tau{}^2)$ and $O(\tilde{a}_\tau \tilde{d}_\tau)$ were neglected. Deviations of the τ dipole moments from the SM values can be determined, possibly down to the level of $O(10^{-3})$ comparing the SM prediction for the differential rate in

Table 1. Branching ratios of radiative τ leptonic decays for a minimum photon energy $\omega_0 = 10$ MeV. Inclusive ($\mathcal{B}^{\text{Inc}}$) and exclusive ($\mathcal{B}^{\text{Exc}}$) predictions are separated into LO contributions ($\mathcal{B}_{\text{LO}}$) and NLO corrections ($\mathcal{B}^{\text{Inc/Exc}}_{\text{NLO}}$). Uncertainties were estimated for uncomputed NNLO corrections (N), numerical errors (n), and the experimental errors of the lifetimes (τ). The first two types of errors were combined to provide the final theoretical uncertainty (th). The last line reports the experimental measurement of ref. [9].

	$\tau \to e\bar{\nu}\nu\gamma$	$\tau \to \mu\bar{\nu}\nu\gamma$
$\mathcal{B}_{\text{LO}}$	1.834×10^{-2}	3.663×10^{-3}
$\mathcal{B}^{\text{Inc}}_{\text{NLO}}$	$-1.06\,(1)_n(10)_N \times 10^{-3}$	$-5.8\,(1)_n(2)_N \times 10^{-5}$
$\mathcal{B}^{\text{Exc}}_{\text{NLO}}$	$-1.89\,(1)_n(19)_N \times 10^{-3}$	$-9.1\,(1)_n(3)_N \times 10^{-5}$
$\mathcal{B}^{\text{Inc}}$	$1.728\,(10)_{\text{th}}(3)_\tau \times 10^{-2}$	$3.605\,(2)_{\text{th}}(6)_\tau \times 10^{-3}$
$\mathcal{B}^{\text{Exc}}$	$1.645\,(19)_{\text{th}}(3)_\tau \times 10^{-2}$	$3.572\,(3)_{\text{th}}(6)_\tau \times 10^{-3}$
$\mathcal{B}_{\text{EXP}}$	$1.847\,(15)_{\text{st}}(52)_{\text{sy}} \times 10^{-2}$	$3.69\,(3)_{\text{st}}(10)_{\text{sy}} \times 10^{-3}$

eq. (15), modified by the terms G_a, J_a, K_a and L_d, with sufficiently precise data.

The possibility to set bounds on $\tilde{a}_\tau$ via the radiative leptonic τ decays in (14) was suggested long ago in Ref. [60]. In that article the authors proposed to take advantage of a radiation zero of the LO differential decay rate in (15) which occurs when, in the τ rest frame, the final lepton l and the photon are back-to-back, and l has maximal energy. Since a non-standard contribution to a_τ spoils this radiation zero, precise measurements of this phase-space region could be used to set bounds on its value. However, studies with Belle data show no significant improvement of the existing limits: the $\tilde{a}_\tau$ upper limit (UL) that can be achieved with the whole Belle statistics, about 0.9×10^9 τ pairs, is only UL($\tilde{a}_\tau$) $\simeq 2$ [61].

A more powerful method to extract $\tilde{a}_\tau$ and $\tilde{d}_\tau$ consists in the use of an unbinned maximum likelihood fit of events in the full phase space. In this approach we considered $e^+e^- \to \tau^+\tau^-$ events where both τ leptons decay subsequently into a particular final state: $\tau^\mp$ (signal side) decays to the radiative leptonic mode, the other $\tau^\pm$ (tag side) decays to some well known mode with a large branching fraction. As a tag decay mode we chose $\tau^\pm \to \rho^\pm\nu \to \pi^\pm\pi^0\nu$, which also serves as spin analyser and allows us to be sensitive to the spin dependent part of the differential decay width of the signal decay using effects of spin-spin correlation of the τ leptons [62]. With this technique we analyzed a data sample of ($\ell^\mp\nu\nu\gamma$, $\pi^\pm\pi^0\nu$) events corresponding to the total amount of data available at Belle and the one planned at the Belle II experiment.

The feasibility study shows that no improvement is expected from Belle data. However the experimental sensitivity on $\tilde{a}_\tau$ at the Belle II experiment, $\sigma_{\tilde{a}} = 0.012$ [61], can already be competitive with Delphi results in (11). On the other hand, the expected sensitivity on the τ EDM, $\sigma_{\tilde{d}} = 6.1 \times 10^{-17} e{\cdot}$cm [61], is still worse than the most precise measurement of $\tilde{d}_\tau$ done at Belle in τ pair production [27].

7 Conclusions

The magnetic and electric dipole moments of the τ lepton are largely unknown. We proposed to use radiative τ leptonic decays to measure these electromagnetic properties at B factories. Deviations of the τ dipole moments from the SM predictions can be determined via an effective Lagrangian approach.

We studied at the NLO in the SM the differential decay rates and branching ratios of $\tau \to l\gamma\nu\bar{\nu}\,(l = \mu, e)$ decays. Our prediction for $l = \mu$ agrees within 1.1σ with the recent Babar measurements of $\mathcal{B}(\tau \to \mu\gamma\nu\bar{\nu})$, for a minimum photon energy threshold $\omega_0 = 10$ MeV. On the contrary the measurement of $\mathcal{B}(\tau \to e\gamma\nu\bar{\nu})$ differs from our prediction by 3.5σ. This puzzling discrepancy deserves further researches.

Our feasibility study showed that the measurement of the τ anomalous magnetic moment at Belle II can be already competitive with the current bound from Delphi experiment, while the expected sensitivity to the tau EDM is still worse than the most precise measurement done at Belle.

Acknowledgements

I would like to thank the organizers for the invitation and the opportunity to present these results. I thank S. Eidelman, D. Epifanov and M. Passera for our fruitful and ongoing collaboration. I would like also to thank L. Mercolli for participating in the early stages of this work. This work is supported by the Swiss National Science Foundation.

References

[1] D. Hanneke, S. Fogwell, G. Gabrielse, Phys.Rev.Lett. **100**, 120801 (2008), 0801.1134
[2] G. Bennett et al. (Muon G-2 Collaboration), Phys.Rev. **D73**, 072003 (2006), hep-ex/0602035
[3] F. Jegerlehner (2015), 1511.04473
[4] J. Schwinger, Phys. Rev. **73**, 416 (1948)
[5] L. Michel, Proc. Phys. Soc. **A63**, 514 (1950)
[6] C. Bouchiat, L. Michel, Phys. Rev. **106**, 170 (1957)
[7] T. Kinoshita, A. Sirlin, Phys.Rev. **107**, 593 (1957)
[8] T. Kinoshita, A. Sirlin, Phys. Rev. **108**, 844 (1957)
[9] J.P. Lees et al. (BaBar), Phys. Rev. **D91**, 051103 (2015), 1502.01784
[10] W. Buchmuller, D. Wyler, Nucl.Phys. **B268**, 621 (1986)
[11] B. Grzadkowski, M. Iskrzynski, M. Misiak, J. Rosiek, JHEP **10**, 085 (2010), 1008.4884
[12] M. Passera, Phys.Rev. **D75**, 013002 (2007), hep-ph/0606174
[13] T.V. Kukhto, E.A. Kuraev, Z.K. Silagadze, A. Schiller, Nucl. Phys. **B371**, 567 (1992)
[14] A. Czarnecki, B. Krause, W.J. Marciano, Phys.Rev. **D52**, 2619 (1995), hep-ph/9506256
[15] A. Czarnecki, B. Krause, W.J. Marciano, Phys.Rev.Lett. **76**, 3267 (1996), hep-ph/9512369

[16] S. Eidelman, M. Passera, Mod.Phys.Lett. **A22**, 159 (2007), hep-ph/0701260
[17] B. Krause, Phys.Lett. **B390**, 392 (1997), hep-ph/9607259
[18] C. Jarlskog, Z.Phys. **C29**, 491 (1985)
[19] C. Jarlskog, Phys.Rev.Lett. **55**, 1039 (1985)
[20] F. Hoogeveen, Nucl.Phys. **B341**, 322 (1990)
[21] M. Pospelov, I. Khriplovich, Sov.J.Nucl.Phys. **53**, 638 (1991)
[22] E. Commins, Adv.At.Mol.Opt.Phys. **40**, 1 (1999)
[23] J. Baron et al. (ACME), Science **343**, 269 (2014), 1310.7534
[24] T. Fukuyama, Int.J.Mod.Phys. **A27**, 1230015 (2012), 1201.4252
[25] J. Abdallah et al. (DELPHI Collaboration), Eur.Phys.J. **C35**, 159 (2004), hep-ex/0406010
[26] F. Cornet, J.I. Illana, Phys.Rev. **D53**, 1181 (1996), hep-ph/9503466
[27] K. Inami et al. (Belle Collaboration), Phys.Lett. **B551**, 16 (2003), hep-ex/0210066
[28] W. Bernreuther, O. Nachtmann, P. Overmann, Phys.Rev. **D48**, 78 (1993)
[29] G.A. Gonzalez-Sprinberg, A. Santamaria, J. Vidal, Nucl.Phys. **B582**, 3 (2000), hep-ph/0002203
[30] A. Hayreter, G. Valencia, Phys. Rev. **D88**, 013015 (2013), [Erratum: Phys. Rev.D91,no.9,099902(2015)], 1305.6833
[31] A. Hayreter, G. Valencia, JHEP **07**, 174 (2015), 1505.02176
[32] S. Atag, A.A. Billur, JHEP **11**, 060 (2010), 1005.2841
[33] M.A. Samuel, G. Li, Int.J.Theor.Phys. **33**, 1471 (1994)
[34] F. del Aguila, F. Cornet, J.I. Illana, Phys.Lett. **B271**, 256 (1991)
[35] I.J. Kim, Nucl.Phys. **B229**, 251 (1983)
[36] D. Chen et al. (E761 Collaboration), Phys.Rev.Lett. **69**, 3286 (1992)
[37] M.A. Samuel, G.w. Li, R. Mendel, Phys.Rev.Lett. **67**, 668 (1991)
[38] T. Aushev, W. Bartel, A. Bondar, J. Brodzicka, T. Browder et al. (2010), 1002.5012
[39] J. Bernabeu, G. Gonzalez-Sprinberg, J. Papavassiliou, J. Vidal, Nucl.Phys. **B790**, 160 (2008), 0707.2496
[40] Y. Ohnishi, T. Abe, T. Adachi, K. Akai, Y. Arimoto et al., PTEP **2013**, 03A011 (2013)
[41] H. Koiso, A. Morita, Y. Ohnishi, K. Oide, K. Satoh, Prog. Theor. Exp. Phys. **2013**, 03A009 (2013)
[42] D. Webber et al. (MuLan Collaboration), Phys.Rev.Lett. **106**, 041803 (2011), 1010.0991
[43] T. Aoyama, M. Hayakawa, T. Kinoshita, M. Nio, Phys.Rev.Lett. **109**, 111807 (2012), 1205.5368
[44] T. Aoyama, M. Hayakawa, T. Kinoshita, M. Nio, Phys. Rev. **D91**, 033006 (2015), 1412.8284
[45] A. Ferroglia, C. Greub, A. Sirlin, Z. Zhang, Phys.Rev. **D88**, 033012 (2013), 1307.6900
[46] M. Fael, L. Mercolli, M. Passera, Phys.Rev. **D88**, 093011 (2013), 1310.1081
[47] T. Kinoshita, A. Sirlin, Phys.Rev. **113**, 1652 (1959)
[48] C. Fronsdal, H. Uberall, Phys.Rev. **113**, 654 (1959)
[49] S. Eckstein, R. Pratt, Ann.Phys. **8**, 297 (1959)
[50] Y. Kuno, Y. Okada, Rev.Mod.Phys. **73**, 151 (2001), hep-ph/9909265
[51] M. Fael, L. Mercolli, M. Passera, JHEP **07**, 153 (2015), 1506.03416
[52] A. Fischer, T. Kurosu, F. Savatier, Phys.Rev. **D49**, 3426 (1994)
[53] A. Arbuzov, E. Scherbakova, Phys.Lett. **B597**, 285 (2004), hep-ph/0404094
[54] T. Kinoshita, A. Sirlin, Phys. Rev. Lett. **2**, 177 (1959)
[55] T. Lee, M. Nauenberg, Phys.Rev. **133**, B1549 (1964)
[56] B. Falk, L.M. Sehgal, Phys. Lett. **B325**, 509 (1994)
[57] L.M. Sehgal, Phys. Lett. **B569**, 25 (2003), hep-ph/0306166
[58] V.S. Schulz, L.M. Sehgal, Phys. Lett. **B594**, 153 (2004), hep-ph/0404023
[59] T. Bergfeld et al. (CLEO Collaboration), Phys.Rev.Lett. **84**, 830 (2000), hep-ex/9909050
[60] M. Laursen, M.A. Samuel, A. Sen, Phys.Rev. **D29**, 2652 (1984)
[61] S. Eidelman, D. Epifanov, M. Fael, L. Mercolli, M. Passera (2016), 1601.07987
[62] Y.S. Tsai, Phys.Rev. **D4**, 2821 (1971)